Name: ____ **Rising Grade Level:** _______

Summer Work: Rising Algebra I & Algebra Ia **Due Date:** First day of school

Important Information:

This summer work packet will be graded as your first quiz grade of the semester. The skills that are covered in this assignment are skills that are necessary to have mastered prior to starting Algebra I and Algebra Ia. You are permitted to use a calculator if the page is not marked with the symbol below:

For pages that are marked with the no calculator symbol, it is important that you can complete the calculations without the use of a calculator to maintain your fact fluency and number sense. If you need help, reach out to your teachers on the **Rising Algebra I and Ia- Summer Work Help** page on Google Classroom. New GHS students can add this page to their Google Classroom using the code below.

zjyk6it

To finish your summer work efficiently and effectively, we recommend working on your math packet for about 90 minutes per week. Create a schedule for completing on your summer work for math using the chart below.

	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Time							
Minutes Spent Working							

Have a great summer,
Mrs. Potter and Mrs. Warrington

Topic 1: Factors and Multiples

1.1 Prime Factorization

Because 3 is a factor of 24 and $3 \cdot 8 = 24$, 8 is also a factor of 24. The pair 3,8 is called a **factor pair** of 24. The **prime factorization** of a composite number is the number written as a product of its prime factors. You can use factor pairs and a **factor tree** to help find the prime factorization of a number. The factor tree is complete when only prime factors appear in the product.

Example 1 **A classroom has 42 students. The teacher arranges the students in rows. Each row has the same number of students. How many possible arrangements are there?**

Use the factor pairs of 42 to find the number of arrangements.

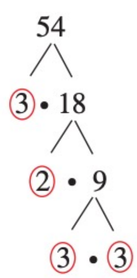
$42 = 1 \bullet 42$	1 row of 42 or 42 rows of 1	$42 = 2 \bullet 21$	2 rows of 21 or 21 rows of 2
$42 = 3 \bullet 14$	3 rows of 14 or 14 rows of 3	$42 = 6 \bullet 7$	6 rows of 7 or 7 rows of 6

▶ There are 8 possible arrangements: 1 row of 42, 42 rows of 1, 2 rows of 21, 21 rows of 2, 3 rows of 14, 14 rows of 3, 6 rows of 7, or 7 rows of 6.

Example 2 **Write the prime factorization of 54.**

Choose any factor pair of 54 to begin the factor tree.

Tree 1

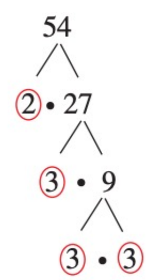


Find a factor pair and draw "branches."

Circle the prime factors as you find them.

Find factors until each branch ends at a prime factor.

Tree 2



$54 = 3 \bullet 2 \bullet 3 \bullet 3$

$54 = 2 \bullet 3 \bullet 3 \bullet 3$

▶ The prime factorization of 54 is $2 \bullet 3 \bullet 3 \bullet 3$, or $2 \bullet 3^3$.

Practice

List the factor pairs of the number.

1. 16

2. 30

3. 63

4. 100

5. 135

6. 275

Write the prime factorization of the number.

7. 24

8. 66

9. 50

10. 98

11. 126

12. 154

1.2 Greatest Common Factor

Factors that are shared by two or more numbers are called **common factors.** The greatest of the common factors is called the **greatest common factor (GCF).** There are several different ways to find the GCF of two or more numbers.

Example 1 **Find the greatest common factor (GCF) of 56 and 104.**

Method 1 List the factors of each number. Then circle the common factors.

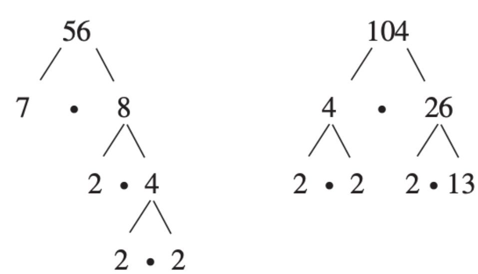
Factors of 56: 1, 2, 4, 7, 8, 14, 28, 56

Factors of 104: 1, 2, 4, 8, 13, 26, 52, 104

The common factors are 1, 2, 4, and 8. The greatest of these common factors is 8.

▶ So, the GCF of 56 and 104 is 8.

Method 2 Make a factor tree for each number.



Write the prime factorization of each number. Then circle the common prime factors. The GCF is the product of the common prime factors.

$$56 = 2 \cdot 2 \cdot 2 \cdot 7$$

$$104 = 2 \cdot 2 \cdot 2 \cdot 13$$

▶ So, the GCF of 56 and 104 is $2 \cdot 2 \cdot 2 = 8$.

Practice

Find the GCF of the numbers using the "L" method or one of the methods shown on the previous page.

1. 30, 45 L-Method Example

$$\begin{array}{c|cc} 5 & 30 & 45 \\ \hline 3 & 6 & 15 \\ \hline & 2 & 5 \end{array}$$

5×3

$\boxed{GCF: 15}$

① Take out common factors one at a time

② Repeat until all common factors have been removed

③ Multiply the numbers on the left

2. 12, 54

3. 16, 96

4. 42, 98

5. 27, 66

6. 50, 160

7. 21, 70

8. 76, 95

9. 60, 84

10. 60, 120, 210 Example

$$\begin{array}{c|ccc} 10 & 60 & 120 & 210 \\ \hline 3 & 6 & 12 & 21 \\ \hline & 2 & 4 & 7 \end{array}$$

10×3

$\boxed{GCF: 30}$

11. 44, 64, 100

12. 15, 28, 70

1.3 Least Common Multiple

Multiples that are shared by two or more numbers are called common multiples. The least of the common multiples is called the least common multiple (LCM). There are several different ways to find the LCM of two or more numbers.

Example 1 **Find the least common multiple (LCM) of 18 and 30.**

Method 1 List the multiples of each number. Then circle the common multiples.

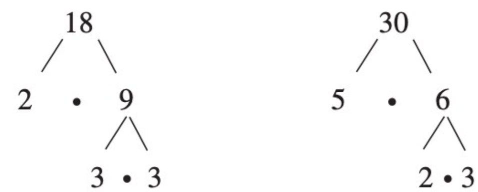
Multiples of 18: 18, 36, 54, 72, 90, 108, 126, 144, 162, 180

Multiples of 30: 30, 60, 90, 120, 150, 180, 210

Some common multiples of 18 and 30 are 90 and 180. The least of these common multiples is 90.

▶ So, the LCM of 18 and 30 is 90.

Method 2 Make a factor tree for each number.



Write the prime factorization of each number. Circle each different factor where it appears the greatest number of times.

$18 = 2 \cdot 3 \cdot 3$ — 2 appears once in both factorizations, so circle it here. 3 appears more often here, so circle all 3s.

$30 = 2 \cdot 3 \cdot 5$ — 5 appears once. Do not circle the 2s or 3s again.

$2 \cdot 3 \cdot 3 \cdot 5 = 90$ — Find the product of the circled factors.

▶ So, the LCM of 18 is 30 is 90.

Practice

Find the LCM of the numbers using the "L" method or one of the methods shown above.

1. 6, 10 L-Method Example

2	6	10
	3 ×	5

① Take out common factors like you did with GCF

② Multiply the side and bottom numbers

$\boxed{\text{LCM: } 30}$

2. 12, 16

3. 15, 25

4. 20, 50

5. 9, 24

6. 10, 22

7. 25, 35

8. 12, 14

9. 10, 18, 20 Example

$$\begin{array}{c|ccc} 2 & 10 & 18 & 20 \\ \hline 5 & 5 & 9 & 10 \\ \hline & 1 & 9 & 2 \end{array}$$

$2 \times 5 \times 1 \times 9 \times 2$

Watch out! If only some of the numbers share a factor, you should still remove it.

$\boxed{\text{LCM: } 180}$

10. 4, 6, 10

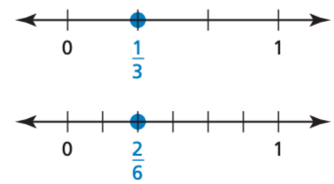
11. 6, 9, 12

12. 16, 24, 30

Topic 2: Fractions, Decimals, and Percents

2.1 Equivalent Fractions and Simplifying Fractions

The number lines to the right show the graphs of two fractions, $\frac{1}{3}$ and $\frac{2}{6}$. These fractions represent the same number. Two fractions that represent the same number are called **equivalent fractions.** To write equivalent fractions, you can multiply or divide the numerator and the denominator by the same nonzero number.



Example 1 **Write two fractions that are equivalent to $\frac{8}{12}$.**

Multiply the numerator and denominator by 2.

$$\frac{8}{12} = \frac{8 \bullet 2}{12 \bullet 2} = \frac{16}{24}$$

Divide the number and denominator by 2.

$$\frac{8}{12} = \frac{8 \div 2}{12 \div 2} = \frac{4}{6}$$

▶ Two equivalent fractions are $\frac{16}{24}$ and $\frac{4}{6}$.

A fraction is in **simplest form** when its numerator and its denominator have no common factors besides 1.

Example 2 **Write the fraction $\frac{18}{24}$ in simplest form.**

Divide the numerator and denominator by 6, the greatest common factor of 18 and 24.

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

▶ $\frac{18}{24}$ in simplest form is $\frac{3}{4}$.

Practice

Write two fractions that are equivalent to the given fraction.

1. $\frac{4}{10}$

2. $\frac{3}{7}$

3. $\frac{10}{15}$

4. $\frac{16}{20}$

5. $\frac{9}{30}$

6. $\frac{1}{8}$

7. $\frac{9}{16}$

8. $\frac{12}{14}$

Write the fraction in simplest form.

9. $\frac{18}{27}$

10. $\frac{3}{18}$

11. $\frac{35}{50}$

12. $\frac{48}{80}$

13. $\frac{24}{63}$

14. $\frac{33}{88}$

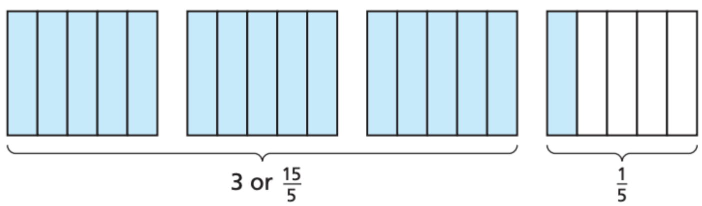
15. $\frac{60}{150}$

16. $\frac{110}{170}$

2.2 Mixed Numbers and Improper Fractions

A **mixed number** is the sum of a whole number and a fraction. An **improper fraction** is a fraction with a numerator that is greater than or equal to the denominator.

The shaded part of the model below represents the mixed number $3\frac{1}{5}$ and the improper fraction $\frac{16}{5}$.



Example 1 **Write $4\frac{5}{8}$ as an improper fraction.**

$$4\frac{5}{8} = 4 + \frac{5}{8}$$ Definition of mixed number

$$= \frac{32}{8} + \frac{5}{8}$$ 1 whole $= \frac{8}{8}$. So, 4 wholes $= \frac{32}{8}$.

$$= \frac{37}{8}$$ Add.

▶ $4\frac{5}{8}$ written as an improper fraction is $\frac{37}{8}$.

Example 2 **Write $\frac{19}{7}$ as a mixed number.**

$$\begin{array}{r} 2 \\ 7\overline{)19} \\ \underline{14} \\ 5 \end{array}$$

Divide the numerator, 19, by the denominator, 7. The quotient is 2.

The remainder is 5. Write the remainder as a fraction, $\frac{\text{remainder}}{\text{divisor}}$.

▶ $\frac{19}{7}$ written as a mixed number is $2\frac{5}{7}$.

Practice

Write the mixed number as an improper fraction.

1. $1\frac{4}{5}$

2. $3\frac{1}{6}$

4. $10\frac{7}{10}$

4. $2\frac{12}{13}$

5. $4\frac{3}{20}$ **6.** $7\frac{6}{7}$ **7.** $6\frac{5}{9}$ **8.** $25\frac{2}{3}$

Write the improper fraction as a mixed number

9. $\frac{9}{2}$ **10.** $\frac{13}{5}$ **11.** $\frac{25}{3}$ **12.** $\frac{31}{9}$

13. $\frac{59}{10}$ **14.** $\frac{43}{4}$ **15.** $\frac{35}{8}$ **16.** $\frac{67}{11}$

2.3 Writing Fractions, Decimals, and Percents

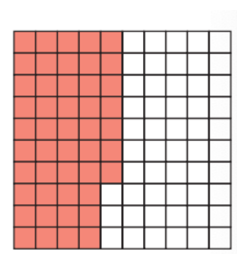
A **percent** is a part-to-whole ratio where the whole is 100. The symbol for percent is %.

In the model, 47 of the 100 squares are shaded. You can write the shaded part of the model as a fraction, a decimal, or a percent.

Fraction: forty-seven out of one hundred, or $\frac{47}{100}$

Decimal: forty-seven hundredths, or 0.47

Percent: forty-seven percent, or 47%



Example 1 **Write the percent or decimal as a fraction.**

a. $86\% = \frac{86}{100} = \frac{43}{50}$ **b.** $125\% = \frac{125}{100} = \frac{5}{4}$, or $1\frac{1}{4}$ **c.** $0.2 = \frac{2}{10} = \frac{1}{5}$

Example 2 **Write the percent or fraction as a decimal.**

a. $19\% = 19. = 0.19$ **b.** $\frac{3}{8} = 3 \div 8 = 0.375$ **c.** $\frac{3}{20} = \frac{3 \cdot 5}{20 \cdot 5} = \frac{15}{100} = 0.15$

Example 3 **Write the decimal or fraction as a percent.**

a. $0.34 = 0.34 = 34\%$ **b.** $0.915 = 0.915 = 91.5\%$ **c.** $\frac{3}{2} = \frac{3 \cdot 50}{2 \cdot 50} = \frac{150}{100} = 150\%$

Practice

Write the percent or decimal as a fraction

1. 0.7 **2.** 0.08 **3.** 1.75 **4.** 0.125

5. 25% **6.** 38% **7.** 1% **8.** 225%

Write the percent or fraction as a decimal.

9. $\frac{3}{4}$ **10.** $\frac{5}{8}$ **11.** $\frac{17}{25}$ **12.** $\frac{101}{200}$

5. 10% **6.** 27% **7.** 100% **8.** 0.8%

Write the decimal or fraction as a percent

17. 0.35 **18.** 0.5 **19.** 1.4 **20.** 0.02

21. $\frac{3}{25}$ **22.** $\frac{17}{20}$ **23.** $\frac{7}{8}$ **24.** $\frac{11}{2}$

2.4 Calculating with Percents

To represent "a is p percent of w," use the **percent proportion** or the **percent equation**.

Percent Proportion	Percent Equation
part $\rightarrow \frac{a}{w} = \frac{p}{100} \leftarrow$ percent whole $\rightarrow$ w	part $\rightarrow a = p \cdot w \leftarrow$ whole $\uparrow$ percent in fraction or decimal form

Example 1 **Answer each question.**

a. What percent of 40 is 18?

percent proportion: $\frac{18}{40} = \frac{p}{100}$

$1800 = 40p$

$45 = p$

percent equation: $18 = p \cdot 40$

$0.45 = p$

So, 45% of 40 is 18.

b. What number is 32% of 75?

$\frac{a}{75} = \frac{32}{100}$

$100a = 2400$

$a = 24$

$a = 0.32 \cdot 75$

$a = 24$

So, 24 is 32% of 75.

c. 125% of what number is 80?

$\frac{80}{w} = \frac{125}{100}$

$8000 = 125w$

$64 = w$

$80 = 1.25 \cdot w$

$64 = w$

So, 125% of 64 is 80.

Practice

Use the percent proportion or percent equation to answer the questions below.

1. 80% of what number is 64?

2. What number is 15% of 130?

3. What percent of 240 is 6?

4. What number is 55% of 94?

5. 3% of what number is 111?

6. What percent of 72 is 64?

7. What percent of 2010 is 94.5?

8. What number is 5% of 6?

9. 20% of what number is 17?

Topic 3: Real Numbers

3.1 Adding and Subtracting Fractions

To add or subtract two fractions with *like denominators*, write the sum or difference of the numerators over the denominator.

Adding or Subtracting Fractions with Like Denominators	
$\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$, where $c \neq 0$	$\frac{a}{c} - \frac{b}{c} = \frac{a-b}{c}$, where $c \neq 0$

Example 1 Find $\frac{7}{12} + \frac{1}{12}$.

$\frac{7}{12} + \frac{1}{12} = \frac{7+1}{12}$ Add the numerators.

$= \frac{8}{12}$, or $\frac{2}{3}$ Simplify.

Example 2 Find $\frac{7}{9} - \frac{2}{9}$.

$\frac{7}{9} - \frac{2}{9} = \frac{7-2}{9}$ Subtract the numerators.

$= \frac{5}{9}$ Simplify.

To add or subtract two fractions with *unlike denominators*, first write equivalent fractions with a common denominator. There are two methods you can use.

Adding or Subtracting Fractions with Unlike Denominators	
Method 1	Multiply the numerator and the denominator of each fraction by the denominator of the other fraction.
Method 2	Use the **least common denominator** (LCD). The LCD of two or more fractions is the least common multiple (LCM) of the denominators.

Example 3 Find $\frac{1}{8} + \frac{5}{6}$.

Method 1: $\frac{1}{8} + \frac{5}{6} = \frac{1 \cdot 6}{8 \cdot 6} + \frac{5 \cdot 8}{6 \cdot 8}$ Rewrite using a common denominator of $8 \cdot 6 = 48$.

$= \frac{6}{48} + \frac{40}{48}$ Multiply.

$= \frac{46}{48}$, or $\frac{23}{24}$ Simplify.

Example 4 Find $5\frac{3}{4} - 1\frac{7}{10}$.

Method 2: Rewrite the difference as $\frac{23}{4} - \frac{17}{10}$.

The LCM of 4 and 10 is 20. So, the LCD is 20.

$\frac{23}{4} - \frac{17}{10} = \frac{23 \cdot 5}{4 \cdot 5} - \frac{17 \cdot 2}{10 \cdot 2}$ Rewrite using the LCD, 20.

$= \frac{115}{20} - \frac{34}{20}$ Multiply.

$= \frac{81}{20}$, or $4\frac{1}{20}$ Simplify.

Practice

Evaluate.

1. $\frac{1}{14} + \frac{5}{14}$

2. $\frac{2}{5} + \frac{1}{5}$

3. $\frac{9}{10} - \frac{1}{10}$

4. $\frac{11}{16} - \frac{3}{16}$

5. $\frac{7}{9} + \frac{2}{3}$

6. $\frac{3}{5} + \frac{4}{7}$

7. $\frac{3}{4} - \frac{1}{6}$

8. $\frac{7}{12} - \frac{5}{9}$

Evaluate.

9. $2\frac{3}{5} + 1\frac{2}{5}$

10. $5\frac{5}{12} + 3\frac{3}{8}$

11. $8\frac{1}{3} - 3\frac{2}{11}$

12. $4\frac{3}{14} - \frac{1}{7}$

3.2 Multiplying and Dividing Fractions.

To multiply two fractions, multiply the numerators and multiply the denominators.

Multiplying Fractions

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}, \text{ where } b, d \neq 0$$

Example 1 **Find $\frac{2}{5} \cdot \frac{3}{8}$.**

$\frac{2}{5} \cdot \frac{3}{8} = \frac{2 \cdot 3}{5 \cdot 8}$ Multiply the numerators. Multiply the denominators.

$= \frac{{}^{1}\cancel{2} \cdot 3}{8 \cdot \cancel{8}_{4}}$ Divide out common factors.

$= \frac{3}{20}$ Simplify.

Example 2 **Find $5\frac{1}{2} \cdot \frac{3}{4}$.**

$5\frac{1}{2} \cdot \frac{3}{4} = \frac{11}{2} \cdot \frac{3}{4}$ Rewrite $5\frac{1}{2}$ as $\frac{11}{2}$.

$= \frac{11 \cdot 3}{2 \cdot 4}$ Multiply the numerators. Multiply the denominators.

$= \frac{33}{8}$, or $4\frac{1}{8}$ Simplify.

Two numbers whose product is 1 are **reciprocals**. To write the reciprocal of a number, write the number as a fraction. Then invert the fraction. Every number except 0 has a reciprocal.

To divide a number by a fraction, multiply the number by the reciprocal of the fraction.

Dividing Fractions

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{a \cdot d}{b \cdot c}, \text{ where } b, c, d \neq 0$$

Example 3 **Find $\frac{3}{7} \div \frac{5}{6}$.**

$\frac{3}{7} \div \frac{5}{6} = \frac{3}{7} \cdot \frac{6}{5}$ Multiply by the reciprocal of $\frac{5}{6}$, which is $\frac{6}{5}$.

$= \frac{3 \cdot 6}{7 \cdot 5}$ Multiply.

$= \frac{18}{35}$ Simplify.

Example 4 **Find $8 \div 2\frac{1}{3}$.**

$8 \div 2\frac{1}{3} = 8 \div \frac{7}{3}$ Rewrite $2\frac{1}{3}$ as $\frac{7}{3}$.

$= 8 \cdot \frac{3}{7}$ Multiply by the reciprocal of $\frac{7}{3}$, which is $\frac{3}{7}$.

$= \frac{8 \cdot 3}{7}$ Multiply.

$= \frac{24}{7}$, or $3\frac{3}{7}$ Simplify.

Practice

Evaluate.

1. $\frac{3}{4} \cdot \frac{1}{6}$

2. $\frac{3}{10} \cdot \frac{2}{3}$

3. $4 \cdot \frac{3}{16}$

4. $3\frac{1}{2} \cdot \frac{6}{7}$

Evaluate.

5. $\frac{1}{6} \div \frac{1}{2}$ **6.** $\frac{7}{8} \div \frac{7}{8}$ **7.** $18 \div \frac{2}{3}$ **8.** $7\frac{1}{2} \div 2\frac{1}{10}$

3.3 Operations with Rational Numbers

To add, subtract, multiply, or divide rational numbers, use the same rules for signs as you used for integers.

Example 1 **Find (a) $-\frac{5}{6} + \frac{2}{3}$ and (b) $7.3 - (-4.8)$.**

a. Write the fractions with the same denominator, then add.

$$-\frac{5}{6} + \frac{2}{3} = -\frac{5}{6} + \frac{4}{6} = \frac{-5 + 4}{6} = \frac{-1}{6} = -\frac{1}{6}$$

b. To subtract a rational number, add its opposite.

$7.3 - (-4.8) = 7.3 + 4.8 = 12.1$ The opposite of -4.8 is 4.8.

Example 2 **Find (a) $2.25 \cdot 8$, (b) $-2.25 \cdot (-8)$, and (c) $-2.25 \cdot 8$.**

a. $2.25 \cdot 8 = 18$ **b.** $-2.25 \cdot (-8) = 18$ **c.** $-2.25 \cdot 8 = -18$

Example 3 **Find $-\frac{4}{9} \div \frac{3}{4}$.**

To divide by a fraction, multiply by its reciprocal.

$$-\frac{4}{9} \div \frac{3}{4} = -\frac{4}{9} \cdot \frac{4}{3} = -\frac{4 \cdot 4}{9 \cdot 3} = -\frac{16}{27}$$

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$.

Practice

Add, subtract, multiply, or divide.

1. $-\frac{1}{6} + \frac{5}{6}$ **2.** $-\frac{7}{10} + \left(-\frac{3}{5}\right)$ **3.** $\frac{4}{9} - \frac{2}{3}$ **4.** $-\frac{5}{6} - \frac{1}{4}$

5. $-\frac{3}{2} \cdot \left(-\frac{1}{8}\right)$ **6.** $-\frac{3}{4} \cdot \frac{7}{12}$ **7.** $\frac{5}{8} \div \left(-\frac{1}{4}\right)$ **8.** $-\frac{4}{7} \div \frac{2}{5}$

3.4 Square Roots

The **square root** of a number is a number that, when multiplied by itself, equals the given number. Every positive number has a positive and negative square root. A. **perfect square** is a number with integers as its square roots.

Example 1 **Find the two square roots of 64.**

$8 \cdot 8 = 64$ and $-8 \cdot (-8) = 64$

▶ So, the square roots of 64 are 8 and -8.

The symbol $\sqrt{}$ is called a **radical sign**. It is used to represent a square root. The number under the radical sign is called the **radicand**.

Example 2 **Find the square root(s).**

a. $\sqrt{49}$

▶ Because $7^2 = 49$, $\sqrt{49} = \sqrt{7^2} = 7$.

b. $-\sqrt{\frac{1}{4}}$

▶ Because $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, $-\sqrt{\frac{1}{4}} = -\sqrt{\left(\frac{1}{2}\right)^2} = -\frac{1}{2}$.

c. $\pm\sqrt{1.21}$

▶ Because $1.1^2 = 1.21$, $\pm\sqrt{1.21} = \pm\sqrt{1.1^2} = \pm 1.1$.

Example 3 **Evaluate $3\sqrt{144} - 10$.**

$$\begin{aligned} 3\sqrt{144} - 10 &= 3(12) - 10 && \text{Evaluate the square root.} \\ &= 36 - 10 && \text{Multiply.} \\ &= 26 && \text{Subtract.} \end{aligned}$$

Practice

Find the square roots.

1. $\sqrt{4}$

2. $-\sqrt{81}$

3. $\pm\sqrt{900}$

4. $\pm\sqrt{\frac{1}{36}}$

5. $\sqrt{\frac{4}{9}}$

6. $-\sqrt{\frac{36}{25}}$

Evaluate the expression.

7. $\sqrt{10 + 6}$

8. $4 - 2\sqrt{9}$

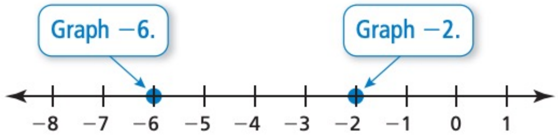
9. $12 - \sqrt{\frac{16}{4}}$

3.5 Comparing and Ordering Real Numbers

There are several ways to compare real numbers. One way is to write the numbers as decimals and use a number line.

Example 1 **Complete the statement with <, >, or =.**

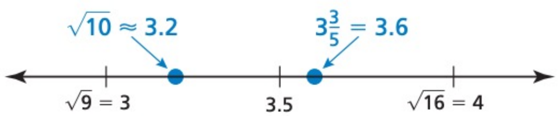
a. -2 ▭ -6



▶ -2 is to the right of -6. So, $-2 > -6$.

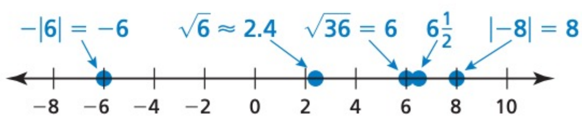
b. $\sqrt{10}$ ▭ $3\frac{3}{5}$

Estimate $\sqrt{10}$ to the nearest tenth. Then graph the numbers on a number line.



▶ $3\frac{3}{5}$ is to the right of $\sqrt{10}$. So, $\sqrt{10} < 3\frac{3}{5}$.

Example 2 **Order the values from least to greatest: $\sqrt{36}$, $|-8|$, $\sqrt{6}$, $6\frac{1}{2}$, $-|6|$.**



▶ So, the order from least to greatest is $-|6|$, $\sqrt{6}$, $\sqrt{36}$, $6\frac{1}{2}$, and $|-8|$.

Practice

Complete the statement with <, >, or =.

1. $-4 \;\square\; -1$

2. $-12 \;\square\; |-13|$

3. $\sqrt{14} \;\square\; 3.75$

4. $2\frac{1}{4} \;\square\; 2.\overline{3}$

Order the values from least to greatest.

5. $3, -|-2|, |-2|, |0|, -1$

6. $\pi,\ 3.14,\ \sqrt{7},\ 2\frac{3}{4},\ \sqrt{4}$

7. $2\pi,\ 5.1\overline{6},\ 5\frac{1}{8},\ \sqrt{25}, 5.25$

8. $|-4^3|, |-9 \cdot 7|, 60, \sqrt{64}$

Topic 4: Algebraic Expressions and Exponents

4.1 Powers and Exponents

A **power** is a product of repeated factors. The **base** is of a power is a common factor. The **exponent** of a power indicates the number of times the base is used as a factor.

base exponent

$$\underbrace{\left(\frac{2}{5}\right)^3}_{\text{power}} = \underbrace{\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}}_{\frac{2}{5} \text{ is used as factor } 3 \text{ times}}$$

Example 1 **Write each product using exponents.**

a. $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9)$

Because -9 is used as a factor 5 times, its exponent is 5.

▶ So, $(-9) \cdot (-9) \cdot (-9) \cdot (-9) \cdot (-9) = (-9)^5$.

b. $\pi \cdot \pi \cdot h \cdot h \cdot h$

Because π is used as a factor 2 times, its exponent is 2. Because h is used as a factor 3 times, its exponent is 3.

▶ So, $\pi \cdot \pi \cdot h \cdot h \cdot h = \pi^2 h^3$.

Example 2 **Evaluate each expression.**

a. $(-5)^4$

$(-5)^4 = (-5) \cdot (-5) \cdot (-5) \cdot (-5)$	Write as repeated multiplication.
$= 625$	Simplify.

b. -5^4

$-5^4 = -(5 \cdot 5 \cdot 5 \cdot 5)$	Write as repeated multiplication.
$= -625$	Simplify.

Practice

Write the product using exponents.

1. $7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$

2. $\left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right) \cdot \left(-\frac{1}{3}\right)$

3. $x \cdot x \cdot y \cdot y \cdot y \cdot y \cdot y$

4. $(-12) \cdot (-12) \cdot v \cdot v \cdot v$

Evaluate the expression.

5. 10^4

6. -15^2

7. $\left(\frac{3}{4}\right)^3$

8. $\left(-\frac{1}{2}\right)^5$

4.2 Properties of Exponents

Product of Powers	Power of a Product	Power of a Power	
$a^m \cdot a^n = a^{m+n}$ Add exponents.	$(ab)^m = a^m b^m$ Find the power of each factor.	$(a^m)^n = a^{mn}$ Multiply exponents.	
Quotient of Powers	**Power of a Quotient**	**Negative Exponent**	**Zero Exponent**
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ Subtract exponents.	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$ Find the power of the numerator and the power of the denominator.	$a^{-n} = \frac{1}{a^n}, a \neq 0$	$a^0 = 1, a \neq 0$

Example 1 Evaluate (a) 4.9^0 and (b) $(-3)^{-4}$.

a. $4.9^0 = 1$ Definition of zero exponent

b. $(-3)^{-4} = \frac{1}{(-3)^4}$ Definition of negative exponent

$= \frac{1}{81}$ Evaluate power.

Example 2 Simplify each expression. Write your answer using only positive exponents.

a. $2^3 \cdot 2^4 = 2^7 = 128$

b. $\frac{5^9}{5^6} = 5^{9-6} = 5^3 = 125$

c. $\frac{12y^0}{x^{-7}} = 12y^0x^7 = 12x^7$

d. $\frac{x^6 \cdot x^2}{x^5} = \frac{x^{6+2}}{x^5} = x^{8-5} = x^3$

e. $(z^4)^2 = z^{4 \cdot 2} = z^8$

f. $(6mn)^3 = 6^3 \cdot m^3 \cdot n^3 = 216m^3n^3$

g. $\left(\frac{y}{3}\right)^4 = \frac{y^4}{3^4} = \frac{y^4}{81}$

h. $\frac{10x^6y^{-2}}{5x^3y} = \frac{10}{5}x^{(6-3)}y^{(-2-1)} = 2x^3y^{-3} = \frac{2x^3}{y^3}$

Practice

Evaluate the expression.

1. $(-9)^0$

2. -8^{-1}

3. 4^3

4. $\frac{-5^0}{3^{-2}}$

Simplify the expression. Write your answer using only positive exponents.

5. $2^9 \cdot 2^{-6}$

6. $-\frac{10^8}{10^{12}}$

7. $y \cdot y^{-5}$

8. $-5x^7 \cdot x^{-11} \cdot 2x^4$

9. $\frac{x^{-2}}{5z^0}$

10. $3x^5 \cdot (-2x)^4$

11. $(5m^2n^1)^3$

12. $\frac{x^7}{x^{-7}}$

13. $(8xy)^2$

14. $(w^5)^{-3}$

4.3 Distributive Property

To multiply a sum or a difference by a number, multiply each number in the sum or difference by the number outside the parentheses, then evaluate.

Distributive Property	
With addition: $5(7 + 3) = 5(7) + 5(3)$	$a(b + c) = a(b) + a(c)$
With subtraction: $5(7 - 3) = 5(7) - 5(3)$	$a(b - c) = a(b) - a(c)$

Example 2 **Simplify each expression.**

a. $6(x + 9)$

$$6(x + 9) = 6(x) + 6(9)$$
$$= 6x + 54$$

b. $10(12 + z + 7)$

$$10(12 + z + 7) = 10(12) + 10(z) + 10(7)$$
$$= 120 + 10z + 70$$
$$= 10z + 190$$

c. $16(8w - 3)$

$$16(8w - 3) = 16(8w) - 16(3)$$
$$= 128w - 48$$

d. $5(4m - 3n - 1)$

$$5(4m - 3\text{n} - 1) = 5(4m) - 5(3n) - 5(1)$$
$$= 20m - 15n - 5$$

Practice

Simplify the expression.

1. $4(y + 7)$

2. $-2(z + 5)$

3. $12(4a + 13)$

4. $9(20 + 17m)$

5. $3(x + 4 + 9)$

6. $6(25 + 6z + 10)$

7. $7(2x + 7 + 9y)$

8. $-4(4r - s + 17)$

9. $1.5(6c + 10d + 3)$

4.4 Evaluating Algebraic Expressions

An **algebraic expression** is an expression that may contain numbers, operations, and one or more symbols. A symbol that represents one or more numbers is called a **variable.** To evaluate an algebraic expression, substitute a number for each variable. Then use the order of operations to find the value of the numerical expression.

Example 1 **Evaluate each expression when $x = 3$.**

a. $5x + 7$

$5x + 7 = 5(3) + 7$ Substitute 3 for x.

$= 15 + 7$ Multiply.

$= 22$ Add.

b. $14 - x^2$

$14 - x^2 = 14 - 3^2$ Substitute 3 for x.

$= 14 - 9$ Evaluate power.

$= 5$ Subtract.

c. $2x^2 - 8x + 4$

$2x^2 - 8x + 4 = 2(3)^2 - 8(3) + 4$ Substitute 3 for x.

$= 2(9) - 8(3) + 4$ Evaluate power.

$= 18 - 24 + 4$ Multiply.

$= -2$ Simplify.

Example 2 **Evaluate each expression when $x = -2$ and $y = 6$.**

a. $7x - 5y$

$7x - 5y = 7(-2) - 5(6)$

$= -14 - 30$

$= -44$

b. $x^2 - 2xy + y^2$

$x^2 - 2xy + y^2 = (-2)^2 - 2(-2)(6) + 6^2$

$= 4 - 2(-2)(6) + 36$

$= 4 - (-24) + 36$

$= 64$

Practice

Evaluate the expression when $x = 2$ and $y = -3$

1. $3x + 10$

2. $14 - 2y$

3. $5 - y^2$

4. $y^2 + 8y - 4$

5. $-3x^2 - x + 7$

6. $2x + 3y$

7. $6y - 5x$

8. $y - x + y^2$

9. $x^2y^2 + xy$

4.5 Simplifying Algebraic Expressions

Parts of an algebraic expression are called *terms*. **Like terms** are terms that have the same variables raised to the same exponents. Constant terms are also like terms.

An algebraic expression in **simplest form** when it has no like terms and no parentheses. To *combine* like terms that have variables, use the Distributive Property to add or subtract the coefficients.

Example 1 **Simplify $8y + 7y$.**

$8y + 7y = (8 + 7)y$	Distributive Property
$= 15y$	Add coefficients.

Example 2 **Simplify $2(x + 5) - 3(x - 2)$.**

$2(x + 5) - 3(x - 2) = 2(x) + 2(5) - 3(x) - 3(-2)$	Distributive Property
$= 2x + 10 - 3x + 6$	Multiply.
$= 2x - 3x + 10 + 6$	Group like terms.
$= -x + 16$	Combine like terms.

Example 3 **Simplify $xy + 3y - 2x + 5y - 3xy$.**

$xy + 3y - 2x + 5y - 3xy = xy - 3xy + 3y + 5y - 2x$	Group like terms.
$= -2xy + 8y - 2x$	Combine like terms.

Practice

Simplify the expression.

1. $7x + 15x$

2. $8y - 14y$

3. $7d + 9 - 5d$

4. $3w + 2(2 - 3w) + 2$

5. $(x + 3) + 3x - 7)$

6. $(5k + 6) + (4k - 8)$

7. $(-7n + 6) + (5n + 15)$

8. $(9z + 12) - (6z + 8)$

9. $s(8 - 2t) + 3t(4 - 2s) + 5t$

4.6 Order of Operations

To evaluate numerical expressions, use a set of rules called the **order of operations.**

Order of Operations
1. Perform operations in **P**arentheses.
2. Evaluate numbers with **E**xponents.
3. **M**ultiply or **D**ivide from left to right.
4. **A**dd or **S**ubtract from left to right.

Example 1 **Evaluate each expression.**

a. $20 - 5 \cdot 6$

$20 - 5 \cdot 6 = 20 - 30$ Multiply 5 and 6.

$= -10$ Subtract 30 from 20.

b. $12 \cdot 3 + 4^2 \div 8$

$12 \cdot 3 + 4^2 \div 8 = 12 \cdot 3 + 16 \div 8$ Evaluate 4^2.

$= 36 + 16 \div 8$ Multiply 12 and 3.

$= 36 + 2$ Divide 16 by 8.

$= 38$ Add 36 and 2.

c. $7(5 - 3) + 6^2 \div (-3)$

$7(5 - 3) + 6^2 \div (-3) = 7(2) + 6^2 \div (-3)$ Perform operation in parentheses.

$= 7(2) + 36 \div (-3)$ Evaluate 6^2.

$= 14 + 36 \div (-3)$ Multiply 7 and 2.

$= 14 + (-12)$ Divide 36 by -3.

$= 2$ Add 14 and -12.

Practice

Evaluate the expression.

1. $1 - 7 + 5^2$

2. $\frac{3-(-9)}{-10+6}$

3. $(12 - 8)^2 \div 2^5$

4. $18 + 9^2 - 7 \cdot (-3)$

5. $6 \div (7 \div 28)$

6. $36 \div (1 - |2 - 7|)$

Topic 5: Solving Equations and Inequalities

5.1 Solving Linear Equations

To determine whether a value is a solution of an equation, substitute the value into the equation.

Example 1 **Determine whether (a) $x = 1$ or (b) $x = -2$ is a solution of $5x - 1 = 4$.**

a. $5x - 1 = -2x + 6$

$5(1) - 1 \stackrel{?}{=} -2(1) + 6$ — Substitute.

$4 = 4$ ✓ — Simplify.

▶ So, $x = 1$ is a solution.

b. $5x - 1 = -2x + 6$

$5(-2) - 1 \stackrel{?}{=} -2(-2) + 6$ — Substitute.

$-11 \neq 10$ ✗ — Simplify.

▶ So, $x = -2$ is *not* a solution.

To solve a linear equation, isolate the variable.

Example 2 **Solve each equation. Check your solution.**

a. $4x - 3 = 13$

$4x - 3 + 3 = 13 + 3$ — Add 3.

$4x = 16$ — Simplify.

$\frac{4x}{4} = \frac{16}{4}$ — Divide by 4.

$x = 4$ — Simplify.

Check

$4x - 3 = 13$

$4(4) - 3 \stackrel{?}{=} 13$

$13 = 13$ ✓

b. $2(y - 8) = y + 6$

$2y - 16 = y + 6$ — Distributive Property

$2y - y - 16 = y - y + 6$ — Subtract y.

$y - 16 = 6$ — Simplify.

$y - 16 + 16 = 6 + 16$ — Add 16.

$y = 22$ — Simplify.

Check

$2(y - 8) = y + 6$

$2(22 - 8) \stackrel{?}{=} 22 + 6$

$28 = 28$ ✓

Practice

Solve the equation. Check your solution.

1. $-\frac{5}{6}t = -15$

2. $x + 5 = -11x$

3. $9(y - 3) = 45$

4. $6n + 3 = -4n + 7$

5. $2c + 5 = 3(c - 8)$

6. $\frac{w-6}{5} = 8$

5.2 Solving Linear Inequalities

To solve an inequality, isolate the variable.

***Note:** Flip the inequality symbol when you multiply or divide by a negative number.

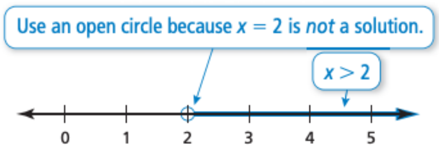
Example 1 **Solve each inequality. Graph the solution.**

a. $x + 1 > 3$

$\underline{-1} \quad \underline{-1}$ Subtract 1 from each side.

$x > 2$ Simplify.

▶ The solution is $x > 2$.

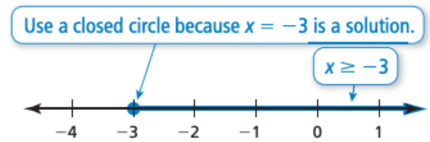


b. $-3x \le 9$

$\frac{-3x}{-3} \ge \frac{9}{-3}$ Divide each side by -3. Reverse the inequality symbol.

$x \ge -3$ Simplify.

▶ The solution is $x \ge -3$.



c. $-25 \ge 9y + 2$

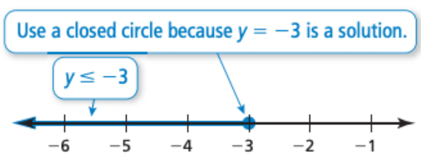
$\underline{-2} \quad \underline{-2}$ Subtract 2 from each side.

$-27 \ge 9y$ Simplify.

$\frac{-27}{9} \ge \frac{9y}{9}$ Divide each side by 9.

$-3 \ge y$ Simplify.

▶ The solution is $y \le -3$.



d. $-\frac{z}{2} + 6 > 10$

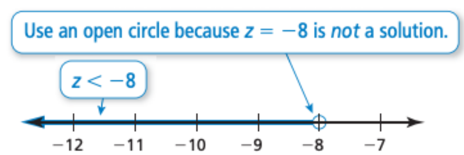
$\underline{-6} \quad \underline{-6}$ Subtract 6 from each side.

$-\frac{z}{2} > 4$ Simplify.

$-\frac{z}{2} \cdot (-2) < 4 \cdot (-2)$ Multiply each side by -2. Reverse the inequality symbol.

$z < -8$ Simplify.

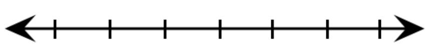
▶ The solution is $z < -8$.



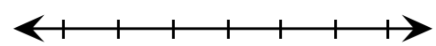
Practice

Solve the inequality. Graph the solution.

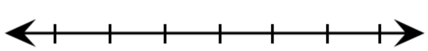
1. $x + 2 > 7$



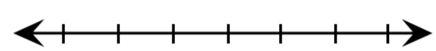
2. $y - 5 \le 8$



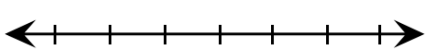
3. $\frac{t}{-3} > 1$



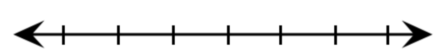
4. $\frac{2s}{5} \le 6$



5. $-2q + 1 \ge 15$



6. $3z - 4 < -1$

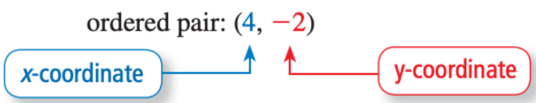


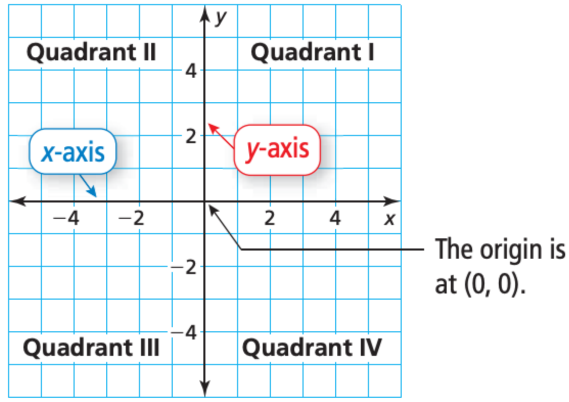
Topic 6: The Coordinate Plane and Slope

6.1 Coordinate Plane

A **coordinate plane** is formed by the intersection of a horizontal number line and a vertical number line. The number lines intersect at the **origin** and separate the coordinate plane into four regions called **quadrants**.

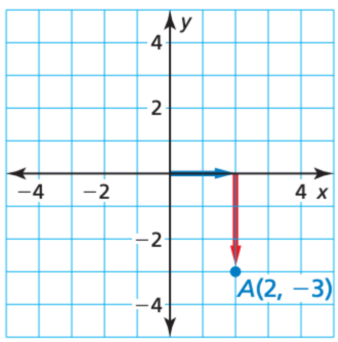
An **ordered pair** is used to locate a point in a coordinate plane.



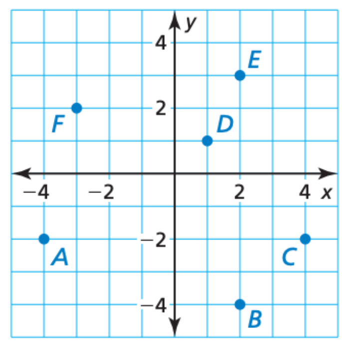


Example 1 **Plot the point $A(2, -3)$ in a coordinate plane. Describe the location of the point.**

Start at the origin. Move 2 units right and 3 units down. Then plot the point. The point is in Quadrant IV.



Example 2 **What ordered pair corresponds to point A?**



Point A is 4 units to the left of the origin and 2 units down. So, the x-coordinate is -4 and the y-coordinate is -2.

▶ The ordered pair $(-4, -2)$ corresponds to point A.

Practice

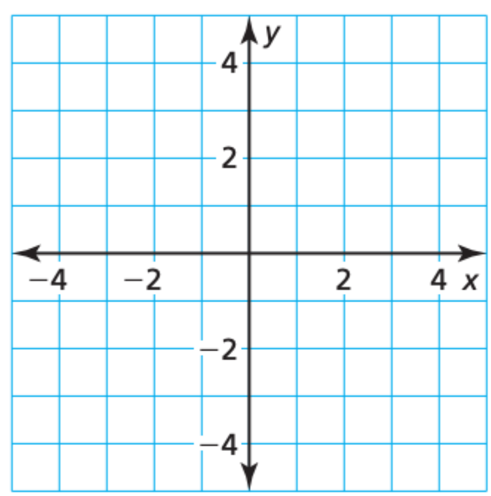
Plot the ordered pair in the coordinate plane. Describe the location of the point.

1. $A(1, 3)$ **2.** $B(-2, 2)$

3. $C(2, -4)$ **4.** $D(1, -1)$

5. $E(-4, -2.5)$ **6.** $F(-3, 0)$

7. $G(0, 1)$ **8.** $H(4, \frac{1}{2})$



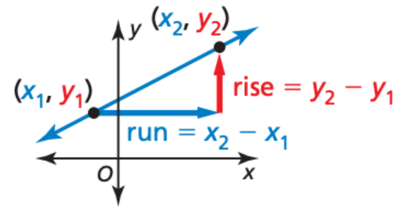
Use the graph in **Example 2** to answer the questions.

9. What ordered pair corresponds to point C?

10. What ordered pair corresponds to point F?

6.2 Slope of a Line

The **slope** of a nonvertical line is the ratio of vertical change (*rise*) to horizontal change (*run*) between any two points on the line. If a line in the coordinate plane passes through points (x_1, y_1) and (x_2, y_2), then the slope m is



$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

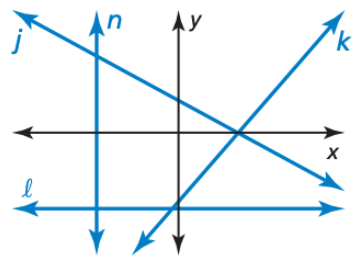
Slopes of Lines in the Coordinate Plane

Negative slope: falls from left to right, as in line j

Positive slope: rises from left to right, as in line k

Zero slope (slope of 0): horizontal, as in line ℓ

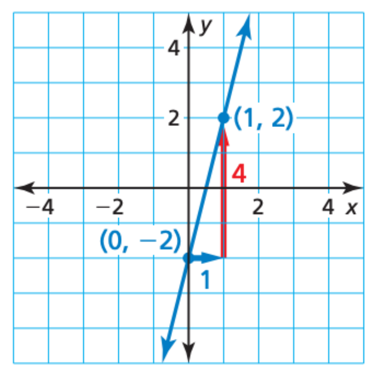
Undefined slope: vertical, as in line n



Example 1 **Find the slope of the line shown.**

Let $(x_1, y_1) = (0, -2)$ and $(x_2, y_2) = (1, 2)$.

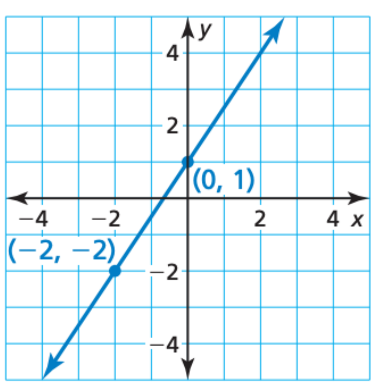
$$\begin{aligned} \text{slope} &= \frac{y_2 - y_1}{x_2 - x_1} && \text{Write formula for slope.} \\ &= \frac{2 - (-2)}{1 - 0} && \text{Substitute.} \\ &= 4 && \text{Simplify.} \end{aligned}$$



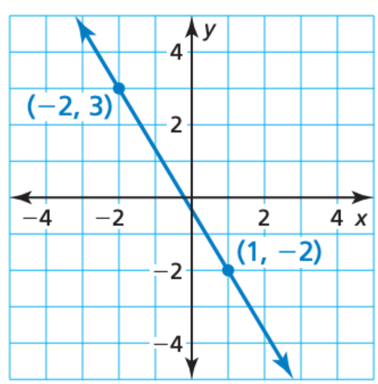
Practice

Find the slope of the line.

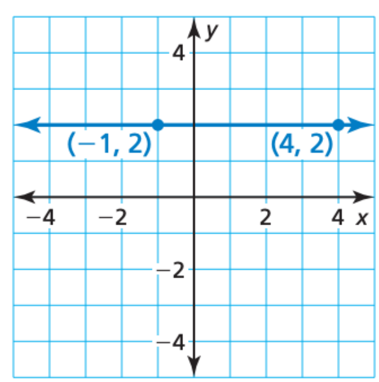
1.



2.



3.



Find the slope of the line through the given points.

4. $(1,2),\ (4,5)$ **5.** $(0,1),\ (3,-3)$ **6.** $(1,2),\ (4,7)$

7. $(-2,5),\ (6,1)$ **8.** $(0,0),\ (3,-9)$ **9.** $(5,0),\ (7,8)$

6.3 Transformations

Translations and Reflections

A **transformation** changes a figure into another figure. The new figure is called the **image**.

A **translation** is a transformation in which a figure slides but does not turn. Every point of the figure moves the same distance and in the same direction. Translating a figure a units horizontally and b units vertically in a coordinate plane changes the coordinates of the figure as follows.

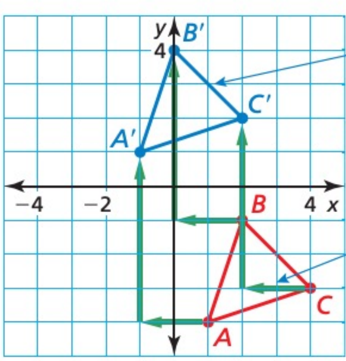
$$(x, y) \rightarrow (x + a, y + b)$$

A **reflection** is a transformation in which a figure is reflected in a line called the **line of reflection**. A reflection creates a mirror image of the original figure. Reflecting a figure in the x-axis or the y-axis changes the coordinates of the figure as follows.

x-axis: $(x, y) \rightarrow (x, -y)$ **y-axis:** $(x, y) \rightarrow (-x, y)$

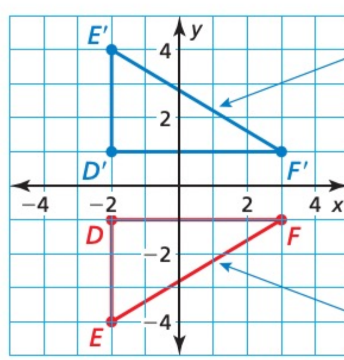
In a translation or reflection, the original figure and its image are congruent.

Example 1 **Translate the red triangle 2 units left and 5 units up. What are the coordinates of the image?**



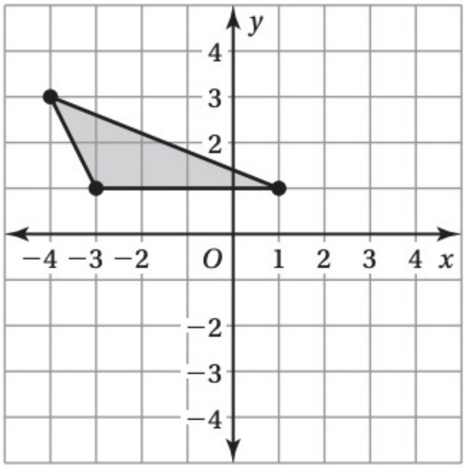
▶ The coordinates of the image are $A'(-1, 1)$, $B'(0, 4)$, and $C'(2, 2)$.

Example 2 **Reflect the red triangle in the x-axis. What are the coordinates of the image?**

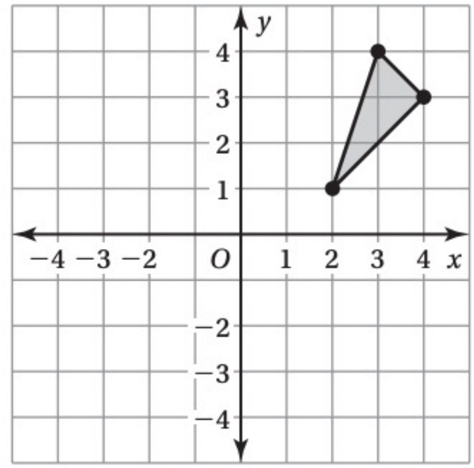


▶ The coordinates of the image are $D'(-2, 1)$, $E'(-2, 4)$, and $F'(3, 1)$.

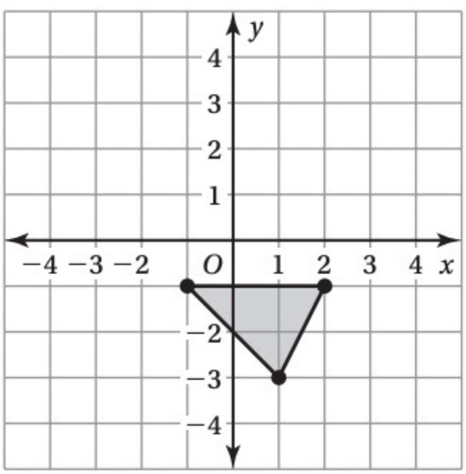
1. Translate the figure 2 units right and 2 units down.



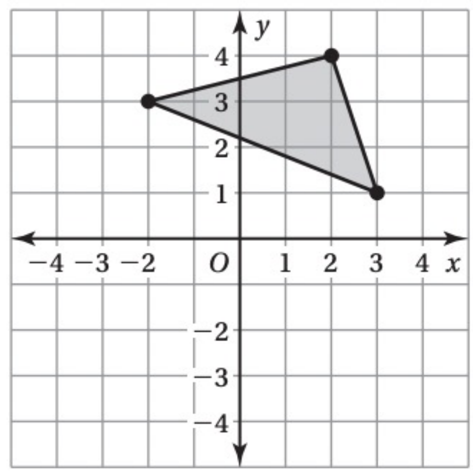
2. Reflect the figure across the y-axis.



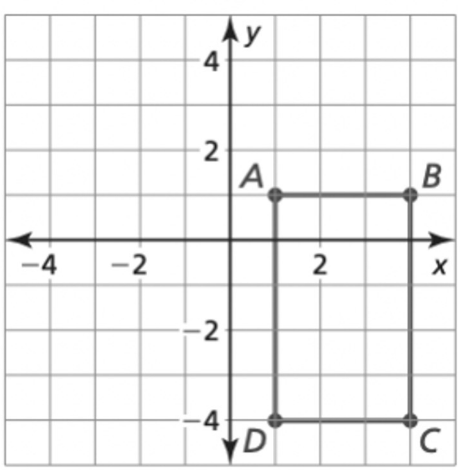
3. Translate the figure 2 units right and 2 units up.



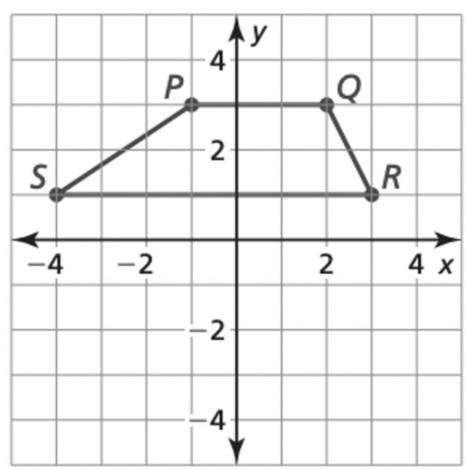
4. Reflect the figure across the x-axis.



5. Translate the rectangle 2 units left and 3 units up.



6. Reflect the trapezoid across the x-axis.



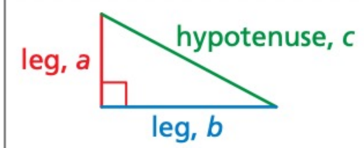
Topic 7: Geometry

7.1 Pythagorean Theorem

In a right triangle, the **hypotenuse** is the side opposite the right angle. The **legs** are the two sides that form the right angle.

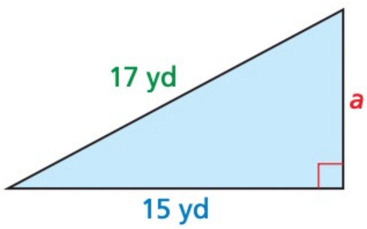
The **Pythagorean Theorem** states that in any right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

Pythagorean Theorem



$$a^2 + b^2 = c^2$$

Example 1 **Find the missing length of the triangle.**



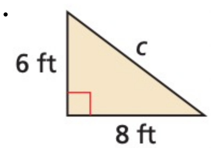
$a^2 + b^2 = c^2$	Write the Pythagorean Theorem.
$a^2 + 15^2 = 17^2$	Substitute 15 for b and 17 for c.
$a^2 + 225 = 289$	Evaluate powers.
$a^2 = 64$	Subtract 225 from each side.
$a = 8$	Take positive square root of each side.

▶ The missing length is 8 yards.

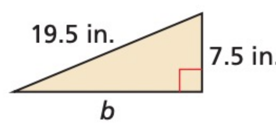
Practice

Find the length of the missing leg. Round to the nearest tenths.

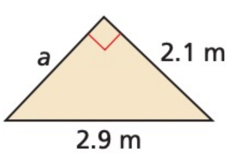
1.



2.



3.



4. $a = 15,\ b = 20,\ c = ?$

5. $a = 11,\ b = ?,\ c = 61$

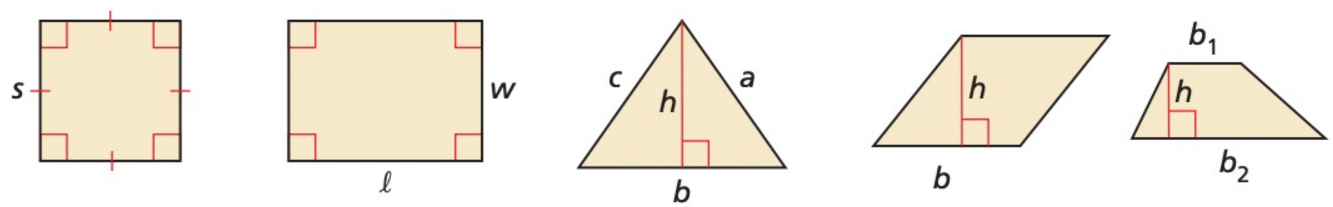
6. $a = ?,\ b = 16,\ c = 34$

7.2a Perimeter and Area of Polygons

The **perimeter** P of a figure is the distance around the figure. The **area** A of a figure is the number of square units enclosed by the figure.

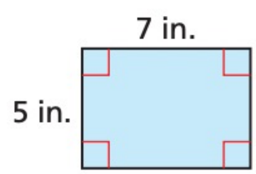
Perimeter and Area

Square	Rectangle	Triangle	Parallelogram	Trapezoid
$P = 4s$	$P = 2\ell + 2w$	$P = a + b + c$	$A = bh$	$A = \frac{1}{2}h(b_1 + b_2)$
$A = s^2$	$A = \ell w$	$A = \frac{1}{2}bh$		



Example 1 **Find the perimeter and area of the figure.**

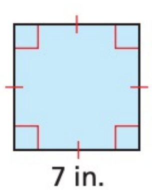
$$P = 2\ell + 2w \qquad A = \ell w$$
$$= 2(7) + 2(5) \qquad = 7(5)$$
$$= 24 \text{ in.} \qquad = 35 \text{ in.}^2$$



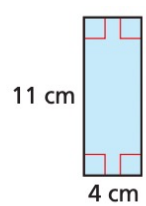
Practice

Find the perimeter and area of each figure.

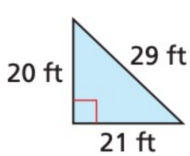
1.



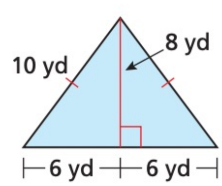
2.



3.

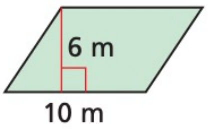


4.

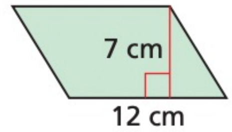


Find the area of each figure.

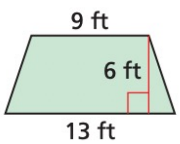
5.



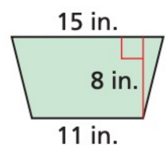
6.



7.

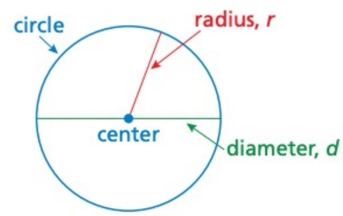


8.



7.2b Area and Circumference of Circles

A **circle** is the set of all points in a plane that are the same distance from a point called the **center**. The distance from the center to any point on the circle is the **radius**. The distance across the circle through the center is the **diameter**. The diameter is twice the radius.



The **circumference** of a circle is the distance around the circle. The ratio $\frac{\text{circumference}}{\text{diameter}}$ is the same for every circle and is represented by the Greek letter π, called **pi**. Pi is an irrational number whose value is approximately 3.14 or $\frac{22}{7}$.

Circumference of a Circle	Area of a Circle
The circumference C of a circle is equal to π times the diameter d or π times twice the radius r. $C = \pi d$ or $C = 2\pi r$	The area A of a circle is the product of π and the square of the radius. $A = \pi r^2$

Example 1 **The diameter of a circle is 8.5 meters. Find the radius.**

$r = \frac{d}{2}$ Radius of a circle

$= \frac{8.5}{2}$ Substitute 8.5 for d.

$= 4.25$ Divide.

▶ The radius is 4.25 meters.

Example 2 **The radius of a circle is $5\frac{3}{4}$ feet. Find the diameter.**

$d = 2r$ Diameter of a circle

$= 2\left(5\frac{3}{4}\right)$ Substitute $5\frac{3}{4}$ for r.

$= 11\frac{1}{2}$

▶ The diameter is $11\frac{1}{2}$ feet.

Example 3 **Find (a) the circumference C and (b) the area A of the circle.**

a. $C = \pi d$

$= \pi(12)$

≈ 37.7

▶ The circumference is about 37.7 yards.

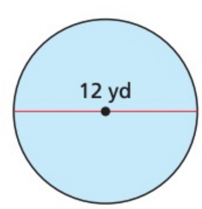
b. $A = \pi r^2$

$= \pi \cdot (6)^2$

$= 36\pi$

≈ 113.1

▶ The area is about 113.1 square yards.



Practice

Find the area of each figure.

1. The radius of a circle is 4.6 millimeters. Find the diameter.

2. The diameter of a circle is $2\frac{1}{4}$. Find the radius.

Find the circumference and area of the circle with the given radius or diameter. Use 3.14 for π.

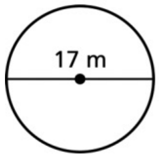
3. $r = 16$ inches.

4. $d = 10$ centimeters

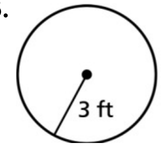
5. $r = 7$ meters.

6. $d = 2.5$ yards.

7.

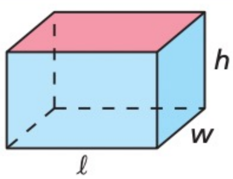
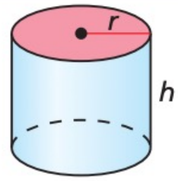
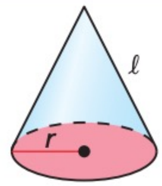
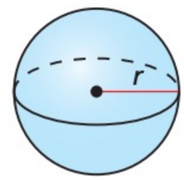


8.

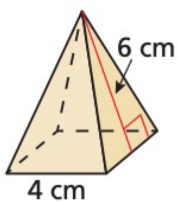


7.3 Surface Area

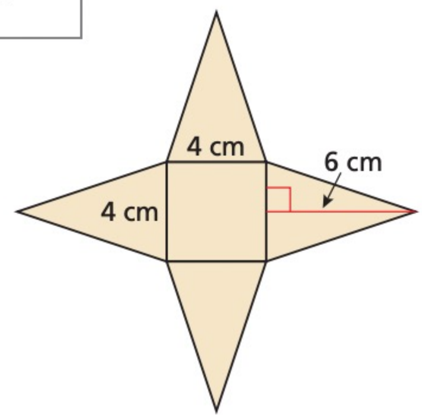
A **solid** is a three-dimensional figure that encloses a space. The **surface area** of a solid is the sum of the areas of all of its faces. Surface area is measured in *square units*. You can use a two-dimensional representation of a solid, called a **net**, to find the surface area of a solid. You can also use the following formulas to find surface areas.

Rectangular Prism	Cylinder	Cone	Sphere
			
$S = 2\ell w + 2\ell h + 2wh$	$S = 2\pi r^2 + 2\pi rh$	$S = \pi r^2 + \pi r\ell$	$S = 4\pi r^2$

Example 1 **Find the surface area of the regular pyramid.**



Draw a net.

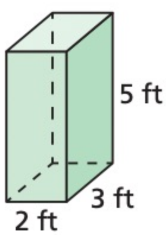


Area of Base	*Area of a Lateral Face*
$4 \cdot 4 = 16$	$\frac{1}{2} \cdot 4 \cdot 6 = 12$

There are four identical lateral faces. So, the surface area is $16 + 4(12) = 64$ square centimeters.

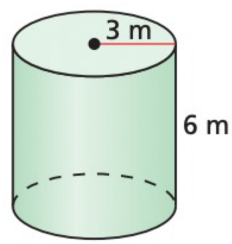
Example 2 **Find the surface area of each solid.**

a.



$$
\begin{aligned}
S &= 2\ell w + 2\ell h + 2wh \\
&= 2(2)(3) + 2(2)(5) + 2(3)(5) \\
&= 12 + 20 + 30 \\
&= 62 \text{ ft}^2
\end{aligned}
$$

b.

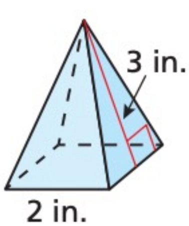


$$
\begin{aligned}
S &= 2\pi r^2 + 2\pi rh \\
&= 2\pi(3)^2 + 2\pi(3)(6) \\
&= 18\pi + 36\pi \\
&= 54\pi \approx 170 \text{ m}^2
\end{aligned}
$$

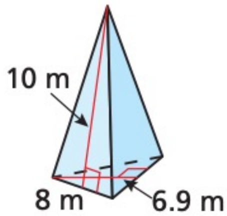
Practice

Find the surface area. Round to the nearest tenths.

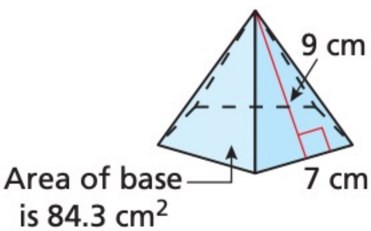
1.



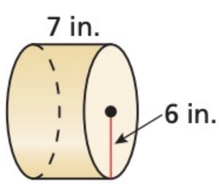
2.



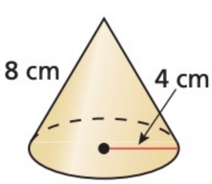
3.



4.

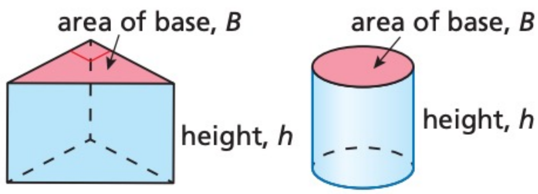
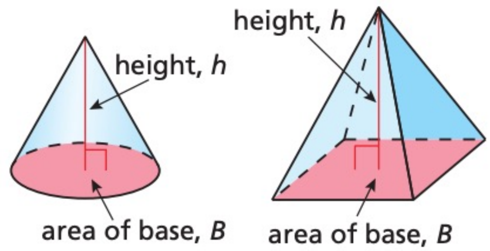
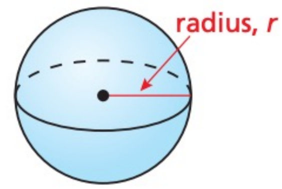


5.



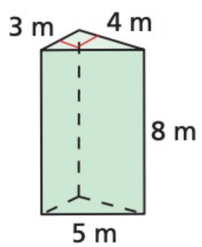
7.4 Volume

A **volume** of a solid is a measure of the amount of space that it occupies. Volume is measured in *cubic units*. You can use the following formulas to find volumes.

Prism and Cylinder	Cone and Pyramid	Sphere
		
$V = Bh$	$V = \frac{1}{3}Bh$	$V = \frac{4}{3}\pi r^3$

Example 1 **Find the volume of each solid.**

a.



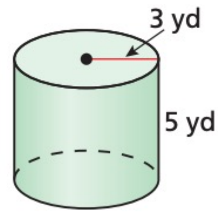
$V = Bh$

$= \frac{1}{2}(3)(4) \cdot 8$

$= 6 \cdot 8$

$= 48 \text{ m}^3$

b.

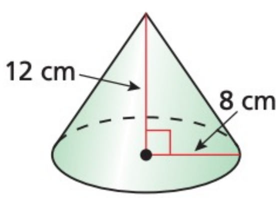


$V = Bh$

$= \pi(3)^2 \cdot 5$

$= 45\pi \approx 141 \text{ yd}^3$

c.

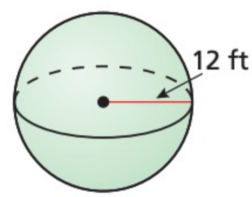


$V = \frac{1}{3}Bh$

$= \frac{1}{3}\pi(8)^2 \cdot 12$

$= 256\pi \approx 804 \text{ cm}^3$

d.



$V = \frac{4}{3}\pi r^3$

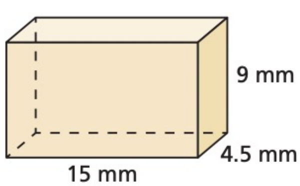
$= \frac{4}{3}\pi(12)^3$

$= 2304\pi \approx 7238 \text{ ft}^3$

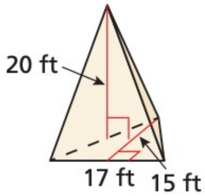
Practice

Find the volume of each figure.

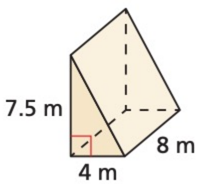
1.



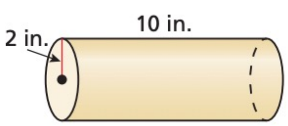
2.



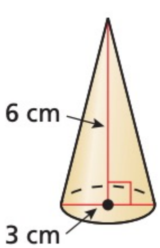
3.



4.



5.



Topic 8: Statistical Measures

8.1 Measures of Center

A **measure of center** is a measure that represents the center, or typical value, of a data set. The *mean*, *median*, and *mode* are measures of center.

Mean	Median	Mode
The **mean** of a numerical data set is the sum of the data divided by the number of data values. The symbol $\bar{x}$ represents the mean. It is read as "*x*-bar."	The **median** of a numerical data set is the middle number when the values are written in numerical order. When a data set has an even number of values, the median is the mean of the two middle values.	The **mode** of a data set is the value or values that occur most often. There may be one mode, no mode, or more than one mode. Mode is the only measure of center that can represent a nonnumerical data set.

Example 1 **The table shows the sizes (in kilobytes) of emails in your inbox.**

a. Find the mean, median, and model of the email sizes.

b. Which measure of center best represent the data? Explain.

Email Sizes (kilobytes)				
1.5	13	1.8	1.9	9.1
2.4	2.8	9.2	2	11
5.6	5	4.9	5.5	11

a. Mean $\bar{x} = \dfrac{1.5 + 13 + 1.8 + \cdots + 5.5 + 11}{15} = 5.78$

Median 1.5, 1.8, 1.9, 2, 2.4, 2.8, 4.9, 5, 5.5, 5.6, 9.1, 9.2, 11, 11, 13 Order the data.

middle value

Mode 1.5, 1.8, 1.9, 2, 2.4, 2.8, 4.9, 5, 5.5, 5.6, 9.1, 9.2, 11, 11, 13 11 occurs most often.

▶ The mean is 5.78 kilobytes, the median is 5 kilobytes, and the mode is 11 kilobytes.

b. The median best represents the data. The mean and mode are both greater than most of the data.

Practice

Find the mean, median, and mode for each data set.

1. $\{35, 44, 40, 35, 54, 50\}$

2. $\{14, 8, 10, 12, 13, 18, 6, 11, 16\}$

3. $\{834, 654, 711, 590, 578, 861, 525\}$

4. $\{4, 8, 5, 6, 4, 5, 4, 2, 6, 5, 4, 3, 5, 4, 6, 5\}$

5. $\{0.6, 1.4, 0.7, 2, 1.5, 1.2, 1.4, 0.9, 0.7, 1.8\}$